**Dutch Mathematical Olympiad 2017**

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1 We consider positive integers written down in the (usual) decimal system. Within such an integer, we number the positions of the digits from left to right, so the leftmost digit (which is never a 0) is at position 1.

An integer is called *even-steven* if each digit at an even position (if there is one) is greater than or equal to its neighbouring digits (if these exist).

An integer is called *oddball* if each digit at an odd position is greater than or equal to its neighbouring digits (if these exist).

For example, 3122 is *oddball* but not *even-steven*, 7 is both *even-steven* and *oddball*, and 123 is neither *even-steven* nor *oddball*.

(a) Prove: every oddball integer greater than 9 can be obtained by adding two *oddball* integers.

(b) Prove: there exists an oddball integer greater than 9 that cannot be obtained by adding two *even-steven* integers.

2 A parallelogram $ABCD$ with $|AD| = |BD|$ has been given. A point E lies on line segment $|BD|$ in such a way that $|AE| = |DE|$. The (extended) line AE intersects line segment BC in F. Line DF is the angle bisector of angle CDE. Determine the size of angle ABD.
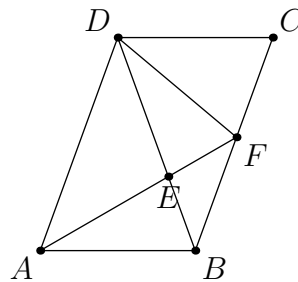
3 Six teams participate in a hockey tournament. Each team plays exactly once against each other team. A team is awarded 3 points for each game they win, 1 point for each draw, and 0 points for each game they lose. After the tournament, a ranking is made. There are no ties in the list. Moreover, it turns out that each team (except the very last team) has exactly 2 points more than the team ranking one place lower.

Prove that the team that finished fourth won exactly two games.

4 If we divide the number 13 by the three numbers $5, 7$, and 9, then these divisions leave remainders: when dividing by 5 the remainder is 3, when dividing by 7 the remainder is 6, and when

dividing by 9 the remainder is 4. If we add these remainders, we obtain $3 + 6 + 4 = 13$, the original number.

(a) Let n be a positive integer and let a and b be two positive integers smaller than n. Prove: if you divide n by a and b, then the sum of the two remainders never equals n.

(b) Determine all integers $n > 229$ having the property that if you divide n by $99, 132$, and 229, the sum of the three remainders is n.

5 The eight points below are the vertices and the midpoints of the sides of a square. We would like to draw a number of circles through the points, in such a way that each pair of points lie on (at least) one of the circles.

Determine the smallest number of circles needed to do this.