Finnish National High School Mathematics Competition 2016

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by parmenides51

1   Which triangles satisfy the equation $\frac{c^2-a^2}{b} + \frac{b^2-c^2}{a} = b-a$ when a, b and c are sides of a triangle?

2   Suppose that y is a positive integer written only with digit 1, in base 9 system. Prove that y is a triangular number, that is, exists positive integer n such that the number y is the sum of the n natural numbers from 1 to n.

3   From the foot of one altitude of the acute triangle, perpendiculars are drawn on the other two sides, that meet the other sides at P and Q. Show that the length of PQ does not depend on which of the three altitudes is selected.

4   How many pairs (a, b) of positive integers a, b solutions of the equation $(4a-b)(4b-a) = 1770^n$ exist, if n is a positive integer?

5   The ruler of Laputa will set up a train network between cities in the state, which satisfies the following conditions:
- *Uniformity*: From one city to another, by train, possibly through exchanges.
- *Prohibition N*: There exist no four cities A, B, C, D such that there are direct routes between A and B, B and C, and C and D, but taking a shortcut is not possible, that is, there are no direct rout between A and C, B and D, or A and D.
In addition, a direct airliner connection will be established exactly between their city pairs, with no direct train connection.
Prove that the airline network is not connected when there is more than one city.