Finnish National High School Mathematics Competition 2017

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by parmenides51

1 By dividing the integer m by the integer $n, 22$ is the quotient and 5 the remainder.
As the division of the remainder with n continues, the new quotient is 0.4 and the new remainder is 0.2.
Find m and n.

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2 Determine $x^2 + y^2$ and $x^4 + y^4$, when $x^3 + y^3 = 2$ and $x + y = 1$

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3 Consider positive integers m and n for which $m > n$ and the number $22220038^m - 22220038^n$ has are eight zeros at the end. Show that $n > 7$.

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4 Let m be a positive integer.
Two players, Axel and Elina play the game HAUKKU (m) proceeds as follows:
Axel starts and the players choose integers alternately. Initially, the set of integers is the set of positive divisors of a positive integer m .The player in turn chooses one of the remaining numbers, and removes that number and all of its multiples from the list of selectable numbers. A player who has to choose number 1, loses. Show that the beginner player, Axel, has a winning strategy in the HAUKKU (m) game for all $m \in Z_+$.

PS. As member Loppukilpailija noted, it should be written $m > 1$, as the statement does not hold for $m = 1$.

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5 Let A and B be two arbitrary points on the circumference of the circle such that AB is not the diameter of the circle. The tangents to the circle drawn at points A and B meet at T. Next, choose the diameter XY so that the segments AX and BY intersect. Let this be the intersection of Q. Prove that the points A, B, and Q lie on a circle with center T.
