Finnish National High School Mathematics Competition 2018

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by parmenides51

1 Eve and Martti have a whole number of euros.
Martti said to Eve: "If you give give me three euros, so I have n times the money compared to you. " Eve in turn said to Martti: "If you give me n euros then I have triple the amount of money compared to you" . Suppose, that both claims are valid.
What values can a positive integer n get?

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2 The sides of triangle ABC are $a = |BC|, b = |CA|$ and $c = |AB|$. Points D, E and F are the points on the sides BC, CA and AB such that AD, BE and CF are the angle bisectors of the triangle ABC. Determine the lengths of the segments AD, BE, and CF in terms of a, b, and c.

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3 The chords AB and CD of a circle intersect at M, which is the midpoint of the chord PQ. The points X and Y are the intersections of the segments AD and PQ, respectively, and BC and PQ, respectively. Show that M is the midpoint of XY.

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4 Define $f : \mathbb{Z}_+ \to \mathbb{Z}_+$ such that $f(1) = 1$ and $f(n)$ is the greatest prime divisor of n for $n > 1$.
Aino and Väinö play a game, where each player has a pile of stones. On each turn the player to turn with m stones in his pile may remove at most $f(m)$ stones from the opponent's pile, but must remove at least one stone. (The own pile stays unchanged.) The first player to clear the opponent's pile wins the game. Prove that there exists a positive integer n such that Aino loses, when both players play optimally, Aino starts, and initially both players have n stones.

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5 Solve the diophantine equation $x^{2018} - y^{2018} = (xy)^{2017}$ when x and y are non-negative integers.
