

Math Prize for Girls Olympiad 2010

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by Ravi B

1 Let S be a set of 100 integers. Suppose that for all positive integers x and y (possibly equal) such that $x + y$ is in S , either x or y (or both) is in S . Prove that the sum of the numbers in S is at most 10,000.

2 Prove that for every positive integer n , there exist integers a and b such that $4a^2 + 9b^2 - 1$ is divisible by n .

3 Let p and q be integers such that q is nonzero. Prove that

$$\left| \frac{p}{q} - \sqrt{7} \right| \geq \frac{24 - 9\sqrt{7}}{q^2}.$$

4 Let S be a set of n points in the coordinate plane. Say that a pair of points is *aligned* if the two points have the same x -coordinate or y -coordinate. Prove that S can be partitioned into disjoint subsets such that (a) each of these subsets is a collinear set of points, and (b) at most $n^{3/2}$ unordered pairs of distinct points in S are aligned but not in the same subset.
