

**Math Prize for Girls Olympiad 2016**

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by Ravi B

- 1 Triangle  $T_1$  has sides of length  $a_1, b_1,$  and  $c_1$ ; its area is  $K_1$ . Triangle  $T_2$  has sides of length  $a_2, b_2,$  and  $c_2$ ; its area is  $K_2$ . Triangle  $T_3$  has sides of length  $a_1 + a_2, b_1 + b_2,$  and  $c_1 + c_2$ ; its area is  $K_3$ .
- (a) Prove that  $K_1^2 + K_2^2 < K_3^2$ .
- (b) Prove that  $\sqrt{K_1} + \sqrt{K_2} \leq \sqrt{K_3}$ .
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- 2 Eve picked some apples, each weighing at most  $\frac{1}{2}$  pound. Her apples weigh a total of  $W$  pounds, where  $W > \frac{1}{3}$ . Prove that she can place all her apples into  $\lceil \frac{3W-1}{2} \rceil$  or fewer baskets, each of which holds up to 1 pound of apples. (The apples are not allowed to be cut into pieces.) Note: If  $x$  is a real number, then  $\lceil x \rceil$  (the ceiling of  $x$ ) is the least integer that is greater than or equal to  $x$ .
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- 3 Let  $n$  be a positive integer. Let  $x_1, x_2, \dots, x_n$  be a sequence of  $n$  real numbers. Say that a sequence  $a_1, a_2, \dots, a_n$  is *unimodular* if each  $a_i$  is  $\pm 1$ . Prove that

$$\sum a_1 a_2 \dots a_n (a_1 x_1 + a_2 x_2 + \dots + a_n x_n)^n = 2^n n! x_1 x_2 \dots x_n,$$

where the sum is over all  $2^n$  unimodular sequences  $a_1, a_2, \dots, a_n$ .

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- 4 Let  $d(n)$  be the number of positive divisors of a positive integer  $n$ . Let  $\mathbb{N}$  be the set of all positive integers. Say that a bijection  $F$  from  $\mathbb{N}$  to  $\mathbb{N}$  is *divisor-friendly* if  $d(F(mn)) = d(F(m))d(F(n))$  for all positive integers  $m$  and  $n$ . (Note: A bijection is a one-to-one, onto function.) Does there exist a divisor-friendly bijection? Prove or disprove.
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