

Dutch Mathematical Olympiad 2002

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by parmenides51

- 1 The sides of a 10 by 10 square $ABCD$ are reflective on the inside. A beam of light enters the square via the vertex A and heads to the point P on CD with $CP = 3$ and $PD = 7$. In P it naturally reflects on the CD side. The light beam can only leave the square via one of the angular points A, B, C or D .
What is the distance that the light beam travels within the square before it leaves the square again?
By which vertex does that happen?

- 2 Determine all triplets (x, y, z) of positive integers with $x \leq y \leq z$ that satisfy $(1 + \frac{1}{x}) (1 + \frac{1}{y}) (1 + \frac{1}{z}) = 3$

- 3 A, B and C are points in the plane with integer coordinates.
The lengths of the sides of triangle ABC are integer numbers.
Prove that the perimeter of the triangle is an even number.

- 4 Five pairs of cartoon characters, Donald and Katrien Duck, Asterix and Obelix, Suske and Wiske, Tom and Jerry, Heer Bommel and Tom Poes, sit around a round table with 10 chairs. The two members of each pair ensure that they sit next to each other. In how many different ways can the ten seats be occupied?
Two ways are different if they cannot be transferred to each other by a rotation.

- 5 In triangle ABC , angle A is twice as large as angle B . $AB = 3$ and $AC = 2$. Calculate BC .