



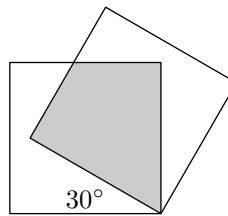
**Dutch Mathematical Olympiad 2003**

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by parmenides51

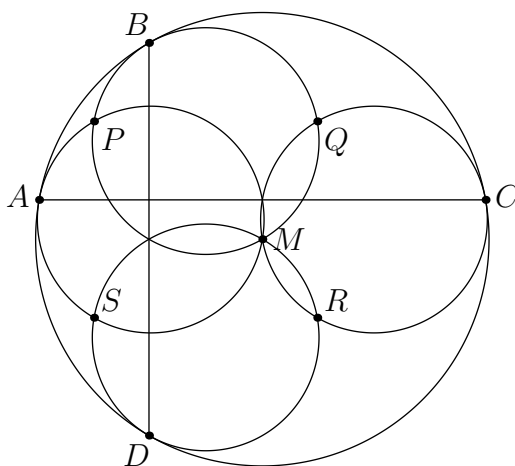
- 1 A Pythagorean triangle is a right triangle whose three sides are integers. The best known example is the triangle with rectangular sides 3 and 4 and hypotenuse 5. Determine all Pythagorean triangles whose area is twice the perimeter.
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- 2 Two squares with side 12 lie exactly on top of each other. One square is rotated around a corner point through an angle of 30 degrees relative to the other square. Determine the area of the common piece of the two squares.



- 3 Determine all positive integers  $n$  that can be written as the product of two consecutive integers and as well as the product of four consecutive integers numbers. In the formula:  $n = a(a + 1) = b(b + 1)(b + 2)(b + 3)$ .
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- 4 In a circle with center  $M$ , two chords  $AC$  and  $BD$  intersect perpendicularly. The circle of diameter  $AM$  intersects the circle of diameter  $BM$  besides  $M$  also in point  $P$ . The circle of diameter  $BM$  intersects the circle with diameter  $CM$  besides  $M$  also in point  $Q$ . The circle of diameter  $CM$  intersects the circle of diameter  $DM$  besides  $M$  also in point  $R$ . The circle of diameter  $DM$  intersects the circle of diameter  $AM$  besides  $M$  also in point  $S$ . Prove that quadrilateral  $PQRS$  is a rectangle.



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- 5 There are a number of cards on a table. A number is written on each card. The "pick and replace" operation involves the following: two random cards are taken from the table and replaced by one new card. If the numbers  $a$  and  $b$  appear on the two packed cards, the number  $a + b + ab$  is set on the new card.
- If we start with ten cards with the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively, what value(s) can the number have that "grab and replace" nine times is on the only card still on the table? Prove your answer
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