



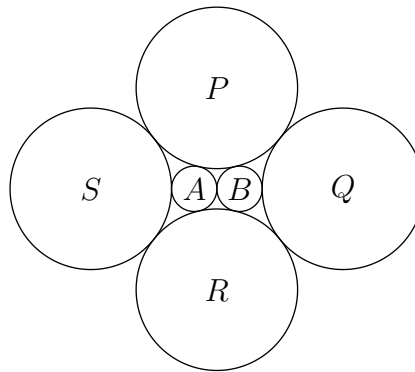
Dutch Mathematical Olympiad 2004

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by parmenides51

- 1 Determine the number of pairs of positive integers (a, b) , with $a \leq b$, for which $\text{lcm}(a, b) = 2004$. $\text{lcm}(a, b)$ means the least common multiple of a and b . Example: $\text{lcm}(18, 24) = 72$.

- 2 Two circles A and B , both with radius 1, touch each other externally. Four circles P, Q, R and S , all four with the same radius r , lie such that P externally touches on A, B, Q and S , Q externally touches on P, B and R , R externally touches on A, B, Q and S , S externally touches on P, A and R . Calculate the length of r .



- 3 Start with a stack of 100 cards. Now repeat the following: choose a stack of at least 2 cards and split them into two smaller piles (at least 1 card of each). Continue this until there are finally 100 stacks of 1 card each. Every time you split a pile into two stacks you get a number of points that is equal to the product of the number of cards in the two new stacks. What is the maximum number of points that you can earn in total?

- 4 Two circles C_1 and C_2 touch each other externally in a point P . At point C_1 there is a point Q such that the tangent line in Q at C_1 intersects the circle C_2 at points A and B . The line QP still intersects C_2 at point C . Prove that triangle ABC is isosceles.

- 5 A right triangle with perpendicular sides a and b and hypotenuse c has the following properties: $a = p^m$ and $b = q^n$ with p and q prime numbers and m and n positive integers, $c = 2k + 1$ with k a positive integer.

Determine all possible values of c and the associated values of a and b .
