Dutch Mathematical Olympiad 2006

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1 A palindrome is a word that doesn't matter if you read it from left to right or from right to left.
Examples: OMO, lepel and parterretrap.
How many palindromes can you make with the five letters a, b, c, d and e under the conditions:
- each letter may appear no more than twice in each palindrome,
- the length of each palindrome is at least 3 letters.
(Any possible combination of letters is considered a word.)

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2 Given is a acute angled triangle ABC. The lengths of the altitudes from A, B and C are successively h_A, h_B and h_C. Inside the triangle is a point P. The distance from P to BC is $1/3h_A$ and the distance from P to AC is $1/4h_B$. Express the distance from P to AB in terms of h_C.

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3 $1+2+3+4+5+6=6+7+8$.
What is the smallest number k greater than 6 for which: $1 + 2 + ... + k = k + (k + 1) + ... + n$, with n an integer greater than k ?

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4 Given is triangle ABC with an inscribed circle with center M and radius r.
The tangent to this circle parallel to BC intersects AC in D and AB in E.
The tangent to this circle parallel to AC intersects AB in F and BC in G.
The tangent to this circle parallel to AB intersects BC in H and AC in K.
Name the centers of the inscribed circles of triangle AED, triangle FBG and triangle KHC successively M_A, M_B, M_C and the rays successively r_A, r_B and r_C.
Prove that $r_A + r_B + r_C = r$.

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5 Player A and player B play the next game on an 8 by 8 square chessboard.
They in turn color a field that is not yet colored. One player uses red and the other blue. Player A starts. The winner is the first person to color the four squares of a square of 2 by 2 squares with his color somewhere on the board.
Prove that player B can always prevent player A from winning.
