6th IGO

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by parmenides51, Dadgarnia

–	Elementary
1	There is a table in the shape of a 8×5 rectangle with four holes on its corners. After shooting a ball from points A, B and C on the shown paths, will the ball fall into any of the holes after 6 reflections? (The ball reflects with the same angle after contacting the table edges.) `http://s5.picofile.com/file/8372960750/E01.png` *Proposed by Hirad Alipanah*
2	As shown in the figure, there are two rectangles $ABCD$ and $PQRD$ with the same area, and with parallel corresponding edges. Let points N, M and T be the midpoints of segments QR, PC and AB, respectively. Prove that points N, M and T lie on the same line. `http://s4.picofile.com/file/8372959484/E02.png` *Proposed by Morteza Saghafian*
3	There are $n>2$ lines on the plane in general position; Meaning any two of them meet, but no three are concurrent. All their intersection points are marked, and then all the lines are removed, but the marked points are remained. It is not known which marked point belongs to which two lines. Is it possible to know which line belongs where, and restore them all? *Proposed by Boris Frenkin - Russia*
4	Quadrilateral $ABCD$ is given such that $$\angle DAC = \angle CAB = 60^\circ,$$ and $$AB = BD - AC.$$ Lines AB and CD intersect each other at point E. Prove that $$\angle ADB = 2\angle BEC.$$ *Proposed by Iman Maghsoudi*

5 For a convex polygon (i.e. all angles less than 180°) call a diagonal *bisector* if its bisects both area and perimeter of the polygon. What is the maximum number of bisector diagonals for a convex pentagon?

Proposed by Morteza Saghafian

– Intermediate

1 Two circles ω_1 and ω_2 with centers O_1 and O_2 respectively intersect each other at points A and B, and point O_1 lies on ω_2. Let P be an arbitrary point lying on ω_1. Lines BP, AP and O_1O_2 cut ω_2 for the second time at points X, Y and C, respectively. Prove that quadrilateral $XPYC$ is a parallelogram.

Proposed by Iman Maghsoudi

2 Find all quadrilaterals $ABCD$ such that all four triangles DAB, CDA, BCD and ABC are similar to one-another.

Proposed by Morteza Saghafian

3 Three circles ω_1, ω_2 and ω_3 pass through one common point, say P. The tangent line to ω_1 at P intersects ω_2 and ω_3 for the second time at points $P_{1,2}$ and $P_{1,3}$, respectively. Points $P_{2,1}$, $P_{2,3}$, $P_{3,1}$ and $P_{3,2}$ are similarly defined. Prove that the perpendicular bisector of segments $P_{1,2}P_{1,3}$, $P_{2,1}P_{2,3}$ and $P_{3,1}P_{3,2}$ are concurrent.

Proposed by Mahdi Etesamifard

4 Let $ABCD$ be a parallelogram and let K be a point on line AD such that $BK = AB$. Suppose that P is an arbitrary point on AB, and the perpendicular bisector of PC intersects the circumcircle of triangle APD at points X, Y. Prove that the circumcircle of triangle ABK passes through the orthocenter of triangle AXY.

Proposed by Iman Maghsoudi

5 Let ABC be a triangle with $\angle A = 60^\circ$. Points E and F are the foot of angle bisectors of vertices B and C respectively. Points P and Q are considered such that quadrilaterals $BFPE$ and $CEQF$ are parallelograms. Prove that $\angle PAQ > 150^\circ$. (Consider the angle PAQ that does not contain side AB of the triangle.)

Proposed by Alireza Dadgarnia

– Advanced

1 Circles ω_1 and ω_2 intersect each other at points A and B. Point C lies on the tangent line from A to ω_1 such that $\angle ABC = 90^\circ$. Arbitrary line ℓ passes through C and cuts ω_2 at points P and

Q. Lines AP and AQ cut ω_1 for the second time at points X and Z respectively. Let Y be the foot of altitude from A to ℓ. Prove that points X, Y and Z are collinear.

Proposed by Iman Maghsoudi

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2 Is it true that in any convex n-gon with $n > 3$, there exists a vertex and a diagonal passing through this vertex such that the angles of this diagonal with both sides adjacent to this vertex are acute?

Proposed by Boris Frenkin - Russia

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3 Circles ω_1 and ω_2 have centres O_1 and O_2, respectively. These two circles intersect at points X and Y. AB is common tangent line of these two circles such that A lies on ω_1 and B lies on ω_2. Let tangents to ω_1 and ω_2 at X intersect O_1O_2 at points K and L, respectively. Suppose that line BL intersects ω_2 for the second time at M and line AK intersects ω_1 for the second time at N. Prove that lines AM, BN and O_1O_2 concur.

Proposed by Dominik Burek - Poland

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4 Given an acute non-isosceles triangle ABC with circumcircle Γ. M is the midpoint of segment BC and N is the midpoint of arc BC of Γ (the one that doesn't contain A). X and Y are points on Γ such that $BX \parallel CY \parallel AM$. Assume there exists point Z on segment BC such that circumcircle of triangle XYZ is tangent to BC. Let ω be the circumcircle of triangle ZMN. Line AM meets ω for the second time at P. Let K be a point on ω such that $KN \parallel AM$, ω_b be a circle that passes through B, X and tangents to BC and ω_c be a circle that passes through C, Y and tangents to BC. Prove that circle with center K and radius KP is tangent to 3 circles ω_b, ω_c and Γ.

Proposed by Tran Quan - Vietnam

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5 Let points A, B and C lie on the parabola Δ such that the point H, orthocenter of triangle ABC, coincides
with the focus of parabola Δ. Prove that by changing the position of points A, B and C on Δ so that the orthocenter remain at H, inradius of triangle ABC remains unchanged.

Proposed by Mahdi Etesamifard
