Serbia Team Selection Test 2018

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– Day 1

1 Prove that there exists infinetly many natural number n such that at least one of the numbers $2^{2^n}+1$ and $2018^{2^n}+1$ is not a prime.

2 Let n be a fixed positive integer and let $x_1, \ldots, x_n$ be positive real numbers. Prove that

$$x_1\left(1-x_1^2\right)+x_2\left(1-(x_1+x_2)^2\right)+\dots+x_n\left(1-(x_1+...+x_n)^2\right)<\frac{2}{3}.$$

3 Ana and Bob are playing the following game.

- First, Bob draws triangle ABC and a point P inside it.
- Then Ana and Bob alternate, starting with Ana, choosing three different permutations σ_1, σ_2 and σ_3 of $\{A,B,C\}$.
- Finally, Ana draw a triangle $V_1V_2V_3$.

For $i=1,2,3$, let ψ_i be the similarity transformation which takes $\sigma_i(A), \sigma_i(B)$ and $\sigma_i(C)$ to V_i, V_{i+1} and X_i respectively (here $V_4=V_1$) where triangle $\Delta V_iV_{i+1}X_i$ lies on the outside of triangle $V_1V_2V_3$. Finally, let $Q_i=\psi_i(P)$. Ana wins if triangles $Q_1Q_2Q_3$ and ABC are similar (in some order of vertices) and Bob wins otherwise. Determine who has the winning strategy.

– Day 2

4 An isosceles trapezium is called *right* if only one pair of its sides are parallel (i.e parallelograms are not right).
A dissection of a rectangle into n (can be different shapes) right isosceles trapeziums is called *strict* if the union of any $i, (2\le i\le n)$ trapeziums in the dissection do not form a right isosceles trapezium.
Prove that for any $n, n\ge 9$ there is a strict dissection of a 2017×2018 rectangle into n right isosceles trapeziums.

Proposed by Bojan Basic

5 Let H be the orthocenter of ABC ,$AB\neq AC$,and let F be a point on circumcircle of ABC such that $\angle AFH=90^\circ .K$ is the symmetric point of H wrt B.Let P be a point such that $\angle PHB=$

$\angle PBC = 90^\circ$,and Q is the foot of B to CP.Prove that HQ is tangent to tge circumcircle of FHK.

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6 For any positive integer n, define

$$c_n = \min_{(z_1,z_2,...,z_n)\in\{-1,1\}^n} |z_1\cdot 1^{2018} + z_2\cdot 2^{2018} + ... + z_n\cdot n^{2018}|.$$

Is the sequence $(c_n)_{n\in\mathbb{Z}^+}$ bounded?
